EECS 455 Problem Set 3

Due: Tuesday, October 6.

1. A communicatyion system uses BPSK modulation to transmit data bits $b_l, l = 0, 1, 2, \ldots$. In the transmitter a sequence of rectangular pulses is mixed to a carrier frequency by multiplying the rectangular pulses by $\sqrt{2P}\cos(2\pi f_1 t)$.

$$s(t) = \sqrt{2P} \sum_{l=0,1,\ldots} b_l p_T(t - lT)\cos(2\pi f_1 t)$$

   At the receiver the received signal is first mixed down to baseband by multiplying by $\sqrt{2/T}\cos(2\pi f_2 t)$ where $f_2 - f_1 = \Delta f$ is the offset of the two oscillators. After the signal is mixed down it is filtered with a matched filter (that is $h(t) = p_T(t)$). The filter is sampled at time $t = iT$ for $i = 1, 2, \ldots$. In addition the signal is mixed down by multiplying by $\sqrt{2/T}\sin(2\pi f_2 t)$. Let $y_c(iT)$ denote the first output and $y_s(iT)$ denote the second output. Then

$$\begin{aligned} y_c(iT) &= \int_{-\infty}^{\infty} s(\tau)\sqrt{\frac{2}{T}}\cos(2\pi f_2 \tau)h(iT - \tau)d\tau \\ y_s(iT) &= -\int_{-\infty}^{\infty} s(\tau)\sqrt{\frac{2}{T}}\sin(2\pi f_2 \tau)h(iT - \tau)d\tau \end{aligned}$$

   (a) Evaluate the outputs $y_c(iT)$ and $y_s(iT)$ in terms of b_{i-1}, $E = PT$, $\Delta f T$ and i. Ignore double frequency terms in evaluating the output. That is, derive an expression for $y_c(iT)$ and $y_s(iT)$. Trig identity $\sin(u) - \sin(v) = 2\cos(\frac{u+v}{2})\sin(\frac{u-v}{2})$.

   (b) Assume you buy two crystal oscillators at a 10MHz nominal frequency that have $\pm$ 10 PPM accuracy. That is, $f_{\text{actual}} = f_{\text{nominal}}(1 \pm 10/10^6)$. Assume that the data rate is 100kbps ($T = 10^{-5}$). Are the double frequency terms negligible? Plot the output of the filters $y_c(iT)$ and $y_s(iT)$ as a function of i for $1 \le i \le 100$. Assume that the data bits are all positive ($b_i = 1, i = 0, 1, 2, \ldots, 500$) and that $E = 1$.

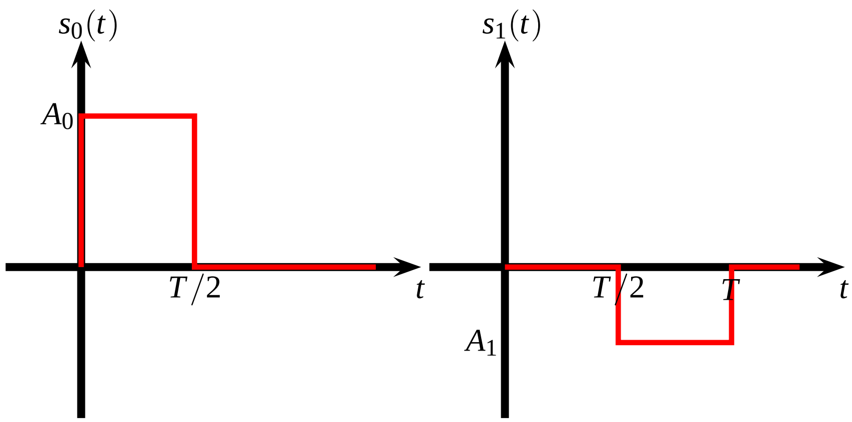
2. Consider a binary communication system that transmits one of two signal $s_0(t)$ and $s_1(t)$ over an additive white Gaussian noise channel (spectral density $N_0/2$) where

$$s_0(t) = A_0 p_{T/2}(t)$$

   and

$$s_1(t) = A_1 p_{T/2}(t - T/2);$$

   that is s_0 is a pulse of amplitude A_0 from 0 to $T/2$ and s_1 is a pulse of amplitude A_1 from $T/2$ to T.



The received signal, $r(t)$, is the transmitted signal with additive white Gaussian noise. The receiver shown below in Figure 1 consist of a filter $h(t)$ which is sampled at time T and a threshold device.

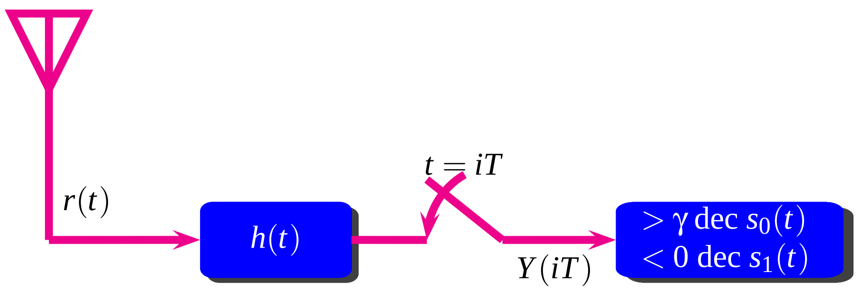


Figure 1: Receiver Structure

(a) If $h(t) = -p_{T/2}(t) + p_{T/2}(t - T/2)$ find the threshold that will minimize the *average* of the error probabilities $P_{e,0}$ and $P_{e,1}$. Find the error average error probability $\overline{P_e} = \pi_0 P_{e,0} + \pi_1 P_{e,1}$. (assume $\pi_0 = \pi_1$)

(b) Find the matched filter for the same system and find the corresponding threshold that minimizes $\overline{P_e}$. Also find $\overline{P_e}$.

(c) If the s_0 is transmitted with probability $\pi_0 = 1/4$ and s_1 is transmitted with probability $\pi_1 = 3/4$ and the filter of part (a) is used find the threshold that minimizes the *average* error probability. What is the average error probability with this threshold?

(d) Repeat part (c) if the matched filter is used.

3. For each signal set below, find the average error probability for binary communication via an AWGN channel (spectral density $N_0/2$). Assume for each signal that the receiver consists of an ideal matched filter, a sampler which samples at an optimal time, and a threshold device with the optimum threshold.

(a)

$$s_0(t) = \begin{cases} A & 0 \le t < T/3 \\ -A & 2T/3 \le t < T \\ 0 & \text{elsewhere} \end{cases}$$

$$s_1(t) = \begin{cases} A & 2T/3 \le t < T \\ -A & T/3 \le t < 2T/3 \\ 0 & \text{elsewhere} \end{cases}$$

(b)

$$s_0(t) = A|\cos(2\pi f_0 t)|p_T(t)$$
$$s_1(t) = A|\sin(2\pi f_0 t)|p_T(t)$$

(c)

$$s_0(t) = A(1+\cos(2\pi f_0 t))p_T(t)$$
$$s_1(t) = A(1+\sin(2\pi f_0 t))p_T(t)$$

In parts (b) and (c), assume $\omega_0 T = 2\pi n$ for some integer n.